is replaced with

$$\bigvee_{i=1}^{N} s_i,$$

that is,

$$s^* = (\bigvee s_i, \bigvee s_i, \cdots, \bigvee s_i, a'_{\max J}, \cdots, a'_{\max N})^T.$$

From the s^* and the A, it is easily verified that the system structure cannot be uniquely determined when criteria 1 and/or 2 are applied. That is, the ill-posed solution is obtained. Q.E.D.

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Object-Handling System for Manual Industry

TOKUJI OKADA

Abstract—**An object-handling system for manual industry is described. This system is designed to have a compact structure and to accomplish multiple prehension and flexible motion for manual tasks. The system has three fingers which have structures similar to those of a human. Namely, these fingers are composed of three, four, and four joints, respectively, and can perform not only such simple motions as bending and extending but also such lateral flexing motions as adduction and abduction. The kinematics of the system is discussed in a rectangular coordinate system. The general solution for the finger joint is obtained by solving a fourth-degree equation. The control mode of each finger joint is suitably changed between position and torque control, and each joint is controlled by a hardware servo system. Cooperative motions among the fingers are easily realized by means of the hardware servo system. In an experiment of swing motion, cooperative motion based on a force control is accomplished between the right and left fingers without dropping the object. Bar turning and sphere turning are accomplished smoothly by a computer control in which control signals for finger joints are generated by interpolating a sequence of set points that has been stored in the computer in the teaching process.**

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I. INTRODUCTION

IN THE FIELD of manual tasks, such as assembling complicated parts, the reduction of human labor is insufficient because object-handling systems for manual tasks lack flexible motion. Flexible movement can be performed only with the dexterity of human fingers, which are hardly replaceable by conventional machines, since traditional machines are designed to excell only in power, speed, and accuracy. In order to achieve advanced automation, a mechanical hand with human-like versatility is required. Although manual tasks may be accomplished by a conventional system combined with various types of machines, in reality, it is difficult to design a system with the versatility that a human has in carrying out various tasks. It is reasonable to attempt the structural and motional capabilities of a human hand in an artificial hand, but it may be impossible to design a hand that behaves equivalently to that of a human, since the system composed of a human hand and brain is a highly developed manipulation system. However, it is considered to be comparatively easy to realize a highly satisfactory mechanical hand by utilizing results [1] obtained from the analysis of the handling functions of a human hand.

Mechanical hands for a manipulation system have been reported. Almost all of these papers are mainly concerned with finger design [2], [3], function [4], mechanisms [5], control circuits [6], and motion control [7]. Artificial hands for physically handicapped persons have also been reported. This type of finger requires high manipulability, sufficient satisfaction of its user's demand, and a pleasing appearance. If these hands, are to perform tasks as flexibly as humans, it is necessary to increase the degrees of freedom of the finger subsystem which directly manipulates an object and to develop a method to control the force of each finger joint. The reasons for these are as follows.

Most artificial fingers lack an ability to manipulate an object flexibly while holding it. Even if the fingers have suitable shape for manual tasks, the position, orientation, and prehensional force of the fingers are not easily changed because of their vise-like or jaw-like structure. And even if the finger is composed of a plural number of joints, the finger lacks the ability to carry out complicated motion because the joints are essentially unified by means of a gear, cam, lever, etc. Furthermore, the fingers cannot follow any rapid and delicate changes in position and force occurring between the fingers and the object, since the torque of the joint is not suitably controlled.

Based on these ideas, an artificial hand for an object-handling system is developed to execute manual tasks [8]. The kinematics of the system and experiments related to its finger subsystem are described. The actual object-handling system does not have a data input device like a TV camera; therefore, the initial state of grasping is assumed as given.

II. Structure of the Mechanical Hand

A. Structure of the Arm Subsystem

Roughly speaking, articulations of a human hand can be classified into those of the shoulder, elbow, wrist, and fingers. The shoulder and elbow can change the fingers' position, whereas the wrist can change their orientation widely. In a task for transferring an object, for example, the shoulder and elbow play an important role. On the other hand, the wrist and fingers are much more active in a manual task. By extracting the essential functions necessary for the manual task from a human hand, the author has devised the configurations of the mechanical hand shown in Fig. 1(b) corresponding to those of a human one shown in Fig. 1(a). The symbols from θ_1 to θ_5 denote the degrees of freedom for the arm, which is composed of the shoulder, elbow, and wrist. The symbols from ϕ_1 to ϕ_{11} denote those for the fingers. Both degrees of freedom θ_3 and θ_4 are constituted by a differential gear mechanism of θ_{dR} (right, as viewed from the hand side) and θ_{dL} (left), as shown in Fig. 1(c).

Essential factors in the determination of the operating position of the fingers are respective lengths between the shoulder and the elbow, the elbow and the wrist, the wrist and the fingers, denoted by l_1, l_2, and l_3 as shown in Fig. 1(b). In our arm subsystem, the wrist is composed of the degrees

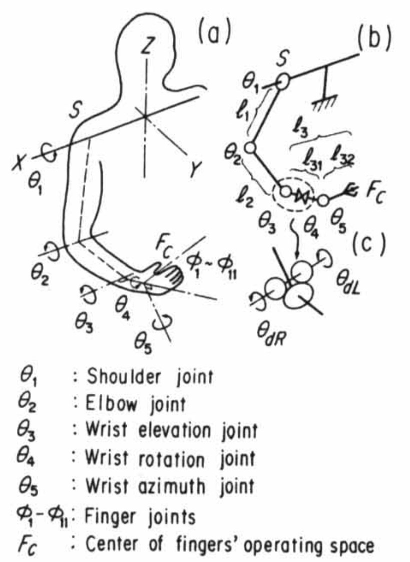


Fig. 1. Structural configurations of human hand and mechanical hand.

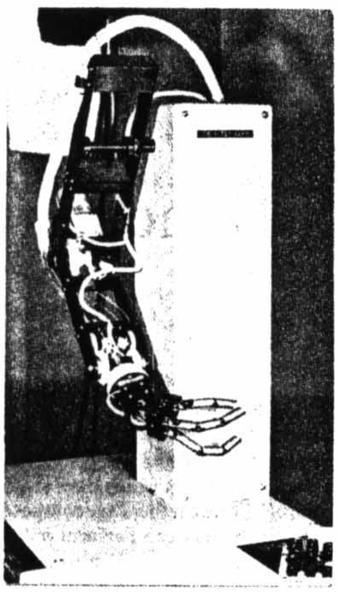


Fig. 2. Photograph of hand.

of freedom θ_3, θ_4 and θ_5. Therefore, the length of l_3 is expressed as $l_{31} + l_{32}$, where l_{31} is the length between the centers of the degrees of freedom θ_3 and θ_5, and l_{32} is the length between the center of the degree of θ_5 and the center of the fingers' operating space. The arm is designed to satisfy the condition $l_1 = l_2 = l_3$, to make a movable area F_c, which represents the center of the fingers' operating space maximum, and to make it possible for the hand to take various orientations at the same location of F_c. A photograph of the mechanical hand is shown in Fig. 2.

B. Structure of the Finger Subsystem

Human fingers can perform actions such as wrapping, pinching, picking, gripping, searching, etc. Similarly, the

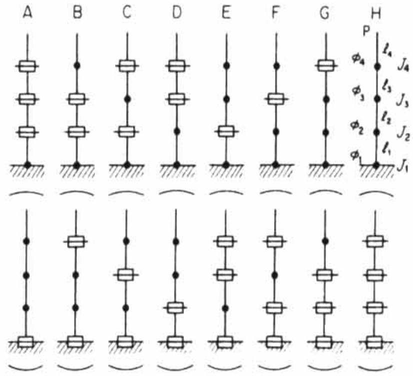


Fig. 3. Constructions of finger with four degrees of freedom.

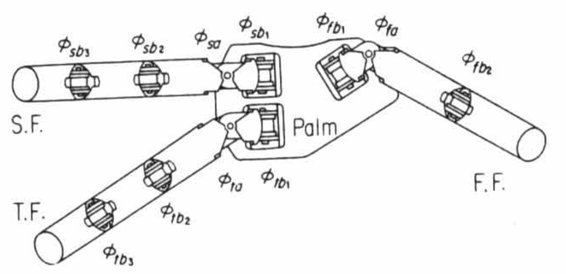


Fig. 4. Structural configuration of finger subsystem.

finger subsystem of the handling system is desired to have satisfactory handling functions for performing various flexible motions required to accomplish these actions. In order to realize these functions, the following three multijointed fingers are adopted for the finger subsystem of the mechanical hand: the first (FF), the second (SF) and the third (TF) fingers corresponding to the thumb, the index, and the middle fingers of a human hand, respectively. The respective numbers of bending and extending joints of these fingers are two, three, and three. Each finger has a joint for lateral flexional motion at its proximal point. Constructions of a finger with four degrees of freedom is shown in Fig. 3. In the figure, J_i, ϕ_i, and l_i are the joint, joint angle, and the length between two neighboring joints, respectively. The rectangle shows the joint which has its rotational axis in the plane of the paper, and the dot shows the joint which has its axis in the plane perpendicular to that plane. Each finger construction in the lower part is similar in kind to its upper part. Therefore, the sixteen finger constructions are of eight different kinds, expressed by the symbols $A \sim H$. In our finger subsystem, the E-type is adopted. The lengths between two neighboring joints of each finger are selected to maximize the rate of enveloping[1] in the grasping state. Structure of the finger subsystem is shown in Fig. 4. Each joint of the finger is driven by cables which run through a coil-like hose so that the cables cannot interfere with complicated finger motion [10]. This driving system protects the cables and makes it unnecessary to set holding points for the guidance of the cables at each joint of the hand. Therefore, the path of power transmission can be selected freely, and the hardware system is compact.

[1] The rate of enveloping is defined to evaluate a finger's adaptability to an object (see [9]).

Fig. 5. View of fingers.

The shape of a finger part is an important element for the determination of a supporting area between the finger and an object to be manipulated, since the smoothness of the manipulation depends on the form of this area. When a plural number of fingers performs three-dimensional motions, the shape of the finger has direct bearing on the smoothness of the task. The finger is designed to have a circular shape, since the circular cross section is the most suitable to suppress the rapid change of the grasping condition which arises during complicated finger motions. The frame of the finger is made of 17-mmϕ free-cutting brass rod, which is bored to be cylindrical.

To make the fingers flexible and to make the finger subsystem more compact, the cables, hoses, and signal lines of sensors are made to pass through the finger tubes [8]. The fingers are shown in Fig. 5. Motors for driving the finger joints are located within a trunk separated from the fingers. The bending angle of each joint and the corresponding torque are indirectly detected from a potentiometer and a value of the motor current in the trunk, respectively. The stranded stainless steel cables connecting each finger joint with the corresponding driving motor are about 170 cm long. The finger subsystem only weighs 240 grams, and it can hold an object of 500 grams.

C. Specifications for Each Degree of Freedom

Each joint of the mechanical hand is driven by a dc motor. The respective angular range of each joint is shown in Table I. The ranges of the arm subsystem are settled to locate the fingers' operating space in the front area of the hand. The spread of the fingers is large as compared with that of a human. Owing to this ability, the system can not only press down on an object from the outside, but also press out from the inside to grasp a hollow object.

The arm is controlled in position in order to extend the fingers' operating space and to help them in manual tasks that are difficult to perform by themselves. On the other hand, the torque of the finger joint is controlled by changing the magnitude of a motor armature current, and the fingers

TABLE I
SPECIFICATIONS OF HANDLING SYSTEM

Freedom		Motion	Angular Range (deg)	Speed (deg/sec)	Gear Ratio	Drive	Power (w)
ARM	θ_1	Shoulder Swing	-45 ~ 45	45	1/208	G	30
	θ_2	Elbow extension	0 ~ 90	45	1/500	G	10
	$\theta_3(\theta_{dR}, \theta_{dL})$ θ_4	Wrist (elevation, rotation)	-180 ~ 180	45	1/397	G,C	2.2
	θ_5	Wrist azimuth	-60 ~ 60	90	1/497	G,C	2.2
F. F.	ϕ_1, ϕ_{fb2}	Bend, Extend	-45 ~ 90	600	1/94.3	G, C	2.2
	ϕ_2, ϕ_{fb1}		-45 ~ 45	500			
	ϕ_3, ϕ_{fa}	Abduct, Adduct	-60 ~ 60	500			
S. F.	ϕ_4, ϕ_{sb3}	Bend, Extend	-45 ~ 90	600			
	ϕ_5, ϕ_{sb2}		-45 ~ 90	600			
	ϕ_6, ϕ_{sb1}		-45 ~ 45	500			
	ϕ_7, ϕ_{sa}	Abduct, Adduct	-60 ~ 45	500			
T. F.	ϕ_8, ϕ_{tb3}	Bend, Extend	-45 ~ 90	600			
	ϕ_9, ϕ_{tb2}		-45 ~ 90	600			
	ϕ_{10}, ϕ_{tb1}		-45 ~ 45	500			
	ϕ_{11}, ϕ_{ta}	Abduct, Adduct	-60 ~ 45	500			

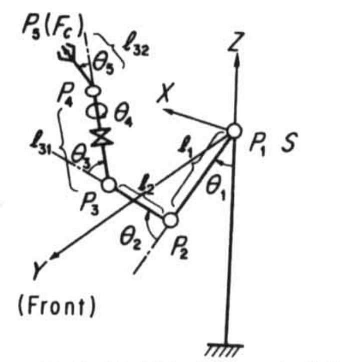


Fig. 6. Variables to be controlled.

are controlled flexibly by suitably changing the mode between position and torque control. Table I shows the electrical and mechanical specifications of the hand. G and C in the column "Drive" denote gears and cables, respectively, as the force transmitting medium, and both θ_{dR} and θ_{dL} denote the rotational angles of the differential gears. The values of θ_3 and θ_4 are determined from the relations of θ_{dR} and θ_{dL}.

III. KINEMATICS OF THE HANDLING SYSTEM

A. Arm Subsystem

As shown in Fig. 6, the position of the center of the fingers' operating space F_c can be expressed in the rectangular coordinate system, where the origin is at the shoulder joint and the X, Y, and Z axes correspond to the side, front, and vertical directions of the hand, respectively. These coordinates, i.e., the position components of the point F_c, are calculated from the following matrix relations:

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = A_0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \tag{1}$$

where A_0 is the matrix obtained by multiplying coordinate transformation matrices $A_1, A_2, \cdots, A_5$ for the respective joints of the arm subsystem in a right-handed sense [11]. That is,

$$A_0 = A_1 A_2 \cdots A_5 = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ T_{41} & T_{42} & T_{43} & T_{44} \end{bmatrix}, \tag{2}$$

where

$$T_{11} = \cos\theta_4 \cdot \cos\theta_5;$$
$$T_{12} = -\cos\theta_4 \cdot \sin\theta_5;$$
$$T_{13} = \sin\theta_4;$$
$$T_{14} = -l_{32}\cos\theta_4 \cdot \sin\theta_5;$$
$$T_{21} = -\sin\theta_4 \cdot \cos\theta_5 \cdot \cos(\theta_1+\theta_2+\theta_3) + \sin\theta_5 \cdot \sin(\theta_1+\theta_2+\theta_3);$$
$$T_{22} = \sin\theta_4 \cdot \sin\theta_5 \cdot \cos(\theta_1+\theta_2+\theta_3) + \cos\theta_5 \cdot \sin(\theta_1+\theta_2+\theta_3);$$
$$T_{23} = \cos(\theta_1+\theta_2+\theta_3) \cdot \cos\theta_4;$$
$$T_{24} = l_{32}\sin\theta_4 \cdot \sin\theta_5 \cdot \cos(\theta_1+\theta_2+\theta_3) + l_{32}\cos\theta_5 \cdot \sin(\theta_1+\theta_2+\theta_3) + l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_{31}\sin(\theta_1+\theta_2+\theta_3);$$
$$T_{31} = -\sin\theta_4 \cdot \cos\theta_5 \cdot \sin(\theta_1+\theta_2+\theta_3) - \sin\theta_5 \cdot \cos(\theta_1+\theta_2+\theta_3);$$
$$T_{32} = \sin\theta_4 \cdot \sin\theta_5 \cdot \sin(\theta_1+\theta_2+\theta_3) - \cos\theta_5 \cdot \cos(\theta_1+\theta_2+\theta_3);$$
$$T_{33} = \sin(\theta_1+\theta_2+\theta_3) \cdot \cos\theta_4;$$
$$T_{34} = l_{32}\sin\theta_4 \cdot \sin\theta_5 \cdot \sin(\theta_1+\theta_2+\theta_3) - l_{32}\cos\theta_5 \cdot \cos(\theta_1+\theta_2+\theta_3) - l_1\cos\theta_1 - l_2\cos(\theta_1+\theta_2) - l_{31}\cos(\theta_1+\theta_2+\theta_3);$$
$$T_{41} = T_{42} = T_{43} = 0;$$
$$T_{44} = 1.$$

When the axial components in a new coordinate system with an origin at P_4 are projected to the old coordinate system with an origin at P_1, direction cosines are given by A_0 col (1, 0, 0, 0), A_0 col (0, 1, 0, 0) and A_0 col (0, 0, 1, 0), where col denotes the column vector. The direction of l_{32} coincides with that of the Y axis in the coordinate system with an origin at P_4. Therefore, the orientation of the hand is given by these direction cosines expressed in the old coordinate system (i.e., the second column components of A_0 except the fourth row component). Although these direction cosines represent the orientation of the finger subsystem, it is not easy to imagine this situation. Therefore, the following representation is adopted to obtain an intuitive comprehension of fingers' orientation. On a rectangular coordinate system, the orientation is represented with the Euler angles

(α, β, γ), where α, β, and γ are the rotational angles about Z axis, X axis, and Y axis in the coordinate system with an origin at P_1. These angles can be expressed by the elements of the matrix A_0 as follows:

$$\alpha = \tan^{-1}(-T_{12}/T_{22}); \quad (3)$$

$$\beta = \sin^{-1} T_{32}; \quad (4)$$

$$\gamma = \tan^{-1}(-T_{31}/T_{33}). \quad (5)$$

On the other hand, the control input for each joint can be calculated from the following equations by using the designated position (X, Y, Z) and orientation (α, β, γ):

$$\theta_4 = \sin^{-1}(\cos\alpha \cdot \sin\gamma + \sin\alpha \cdot \sin\beta \cdot \cos\gamma); \quad (6)$$

$$\theta_5 = \tan^{-1}\left(\frac{\sin\alpha \cdot \cos\beta}{\cos\alpha \cdot \cos\gamma - \sin\alpha \cdot \sin\beta \cdot \sin\gamma}\right);$$

or

$$\sin^{-1}\left(\frac{-X}{l_{32}\cos\theta_4}\right); \quad (7)$$

$$\theta_1 + \theta_2 + \theta_3 = \tan^{-1}\left(\frac{\cos\beta \cdot \cos\gamma}{\sin\alpha \cdot \sin\gamma - \cos\alpha \cdot \sin\beta \cdot \cos\gamma}\right); \quad (8)$$

$$l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) = Y - l_{31}\sin(\theta_1 + \theta_2 + \theta_3) - l_{32}\{\cos\theta_5 \cdot \sin(\theta_1 + \theta_2 + \theta_3) + \sin\theta_4 \cdot \sin\theta_5 \cdot \cos(\theta_1 + \theta_2 + \theta_3)\}; \quad (9)$$

$$l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) = -Z - l_{31}\cos(\theta_1 + \theta_2 + \theta_3) - l_{32}\{\cos\theta_5 \cdot \cos(\theta_1 + \theta_2 + \theta_3) - \sin\theta_4 \cdot \sin\theta_5 \cdot \sin(\theta_1 + \theta_2 + \theta_3)\}. \quad (10)$$

The angular values of θ_4 and θ_5 are determined from (6) and (7). By using the value of $(\theta_1 + \theta_2 + \theta_3)$ obtained from (8), the values of θ_1 and θ_2 are determined by solving the (9) and (10) (in reality, given by a quadratic equation). Then the value of θ_3 is determined. From these calculations, several sets of values for $\theta_1, \theta_2, \cdots, \theta_5$ are obtained. Therefore, the optimal set of arm solutions is found after testing the condition of a designated position and orientation. The values of θ_{dR} and θ_{dL} are calculated from the values of θ_3 and θ_4 uniquely, since

$$\theta_{dR} = \tfrac{1}{2}(2\theta_3 - \theta_4); \quad (11)$$

$$\theta_{dL} = \tfrac{1}{2}(2\theta_3 + \theta_4). \quad (12)$$

B. Finger Subsystem

The kinematics of the finger subsystem can be discussed by using a rectangular coordinate system assigned to each finger joint in the same way as in Section III-A.

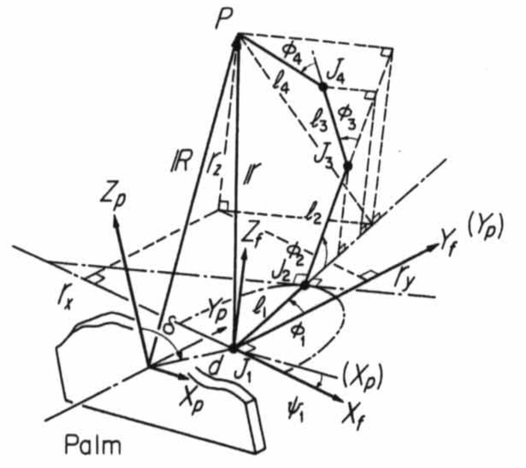


Fig. 7. Geometric configuration of finger.

The geometric configuration of a finger is illustrated in Fig. 7. In the figure, $C_p(X_p, Y_p, Z_p)$ denotes the rectangular coordinate system in the palm area to which the roots of three fingers are attached. The origin of the system C_p is located at the center of the palm. The X_pZ_p-plane is in the palm plane, and the Y_p axis is perpendicular to the palm plane. The positive direction of the axis Y_p is taken to be the direction of the fingers as viewed from the wrist side. The position components of the fingertip along the (X_p, Y_p, Z_p) coordinate axes are (R_x, R_y, R_z). Unit vectors by which the coordinate axes (X_p, Y_p, Z_p) are defined are $(\varepsilon p_1, \varepsilon p_2, \varepsilon p_3)$. Thus the position vector $\boldsymbol{R}$ which points P in the coordinate system C_p is expressed in the form

$$\boldsymbol{R} = R_x \varepsilon p_1 + R_y \varepsilon p_2 + R_z \varepsilon p_3.$$

Further, $C_f(X_f, Y_f, Z_f)$ denotes the rectangular coordinate system having an origin at the root of a finger. The direction of the Y_f axis is the same as that of Y_p. The position components of the fingertip along the (X_f, Y_f, Z_f) coordinate axes are (r_x, r_y, r_z). In the coordinate system C_f, the position vector $\boldsymbol{r}$ which points P is expressed as

$$\boldsymbol{r} = r_x \varepsilon f_1 + r_y \varepsilon f_2 + r_z \varepsilon f_3,$$

where $(\varepsilon f_1, \varepsilon f_2, \varepsilon f_3)$ are unit vectors which define the (X_f, Y_f, Z_f) coordinate axes. Other quantities shown in Fig. 7 have the following definitions:

- i subscript related to a finger joint, 1 is assigned to the root joint;
- l_i length between the ith joint and $(i+1)$th joint;
- ψ_i angle between the ith link's motional plane and the $(i-1)$th link's motional plane;
- ϕ_i angle between the ith link and $(i-1)$th link;
- d distance between the center of the palm and the root of the finger;
- δ angle between the line connecting the center of the palm area with the root of the finger and the Z_p axis, δ is measured clockwise from the Z_p axis.

The components (R_x, R_y, R_z) and (r_x, r_y, r_z) are combined in the matrix relations such that

$$\begin{bmatrix} \cdot & & & R_x \\ & T_p & & R_y \\ & & \cdot & R_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = B_p \begin{bmatrix} \cdot & & & r_x \\ & T_f & & r_y \\ & & \cdot & r_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{13}$$

where B_p is a transformation matrix denoted as

$$\begin{bmatrix} \cos\psi_1 & 0 & \sin\psi_1 & d\sin\delta \\ 0 & 1 & 0 & 0 \\ -\sin\psi_1 & 0 & \cos\psi_1 & d\cos\delta \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

T_p is a rotation matrix about the link l_4 in the coordinate system C_p and also T_f is that in the coordinate system C_f. Then, the transformation equations about the terminal position of the finger are obtained:

$$\begin{aligned} r_x &= R_x \cos\psi_1 - R_z \sin\psi_1 - d \\ &\quad \cdot (\sin\delta \cdot \cos\psi_1 - \cos\delta \cdot \sin\psi_1) \\ r_y &= R_y \\ r_z &= R_x \sin\psi_1 + R_z \cos\psi_1 - d \\ &\quad \cdot (\sin\delta \cdot \sin\psi_1 + \cos\delta \cdot \cos\psi_1). \end{aligned} \tag{14}$$

The parameters (ψ_1, d, δ) are constant values dependent on the mechanical conditions of the handling system. Therefore, the components of the vector r are determined uniquely from the components of the vector R. The elements of T_f are also uniquely determined after matrix multiplication between the matrix inverse of the direction cosine parts of B_p and the matrix of T_p. Thus the position and the orientation designated in the coordinate system C_p are uniquely transformed to those in the coordinate system C_f. In the case where the position and the orientation are given in the coordinate system that has its origin at the shoulder joint, these are also uniquely transformed to those in the coordinate system C_f as far as the control variables about the arm subsystem are fixed. In this case, the transformation matrix B_p is expressed as the result of matrix multiplication (i.e., $A_1 A_2 \cdots A_5 B_p$). Therefore, the essential problem for computer control of manipulator is the determination of joint angles from ϕ_1 to ϕ_4 that locate the fingertip at the point P expressed by the position vector r and that form the orientation expressed by the rotation matrix T_f in the coordinate system C_f.

On the other hand, the components of the terminal position and the rotation matrix about the link l_4 are calculated from a finger configuration. We define these as (r'_x, r'_y, r'_z) and T'_f with a prime in the coordinate system C_f. The equation relating to these values is

$$\begin{bmatrix} \cdot & & & r'_x \\ & T'_f & & r'_y \\ & & \cdot & r'_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = C_1 B_2 C_2 B_3 C_3 B_4 C_4, \tag{15}$$

where B_i is a transformation matrix with respect to ψ_i, and C_i is that with respect to both l_i and ϕ_i in a right-handed sense (see Appendix I). The relations between (r'_x, r'_y, r'_z) and $(\phi_1, \phi_2, \phi_3, \phi_4)$ are dependent on the value of ψ_i. Generally, these are too complicated to determine the finger solution. Therefore, the next three conditions are considered to eliminate the complexity:

$$\begin{aligned} \psi_2 &= \pm\frac{\pi}{2} \\ \psi_3 &= -\psi_2 \\ \psi_4 &= 0. \end{aligned} \tag{16}$$

These conditions are also important in making a finger structure similar to that of a human. The finger which satisfies these conditions is the same as the finger of the E type shown in Fig. 3, which is adopted in our finger subsystem. In these meaningful conditions, the components (r'_x, r'_y, r'_z) are simply expressed with $(\phi_1, \phi_2, \phi_3, \phi_4)$, and the following fundamental equations are obtained under the conditions that $r_x = r'_x$, $r_y = r'_y$, and $r_z = r'_z$:

$$\begin{aligned} r_x &= -(l_1 + M_1 \cos\phi_2)\sin\phi_1 \\ &\quad - \{l_3 \sin\phi_3 + l_4 \sin(\phi_3 + \phi_4)\}\cos\phi_1; \end{aligned} \tag{17}$$

$$\begin{aligned} r_y &= (l_1 + M_1 \cos\phi_2)\cos\phi_1 \\ &\quad - \{l_3 \sin\phi_3 + l_4 \sin(\phi_3 + \phi_4)\}\sin\phi_1; \end{aligned} \tag{18}$$

$$r_z = \operatorname{sign}(\psi_2) M_1 \sin\phi_2, \tag{19}$$

where

$$M_1 = l_2 + l_3 \cos\phi_3 + l_4 \cos(\phi_3 + \phi_4),$$

and the symbol "sign" is the function defined as

$$\operatorname{sign}(\chi) = \begin{cases} 1, & \chi > 0 \\ -1, & \chi < 0. \end{cases}$$

Now we attempt to solve the angular values of ϕ_1, ϕ_2, ϕ_3, and ϕ_4 from (17)–(19). But these values are not uniquely determined because the number of equations is less than that of the unknown quantities. Hence, we assume a reasonable condition that the values of ϕ_3 and ϕ_4 are equal, since these two degrees of freedom are contiguous to each other and are related to the same motional plane. This condition does not limit the operating space of the fingertip. Note that

$$\begin{aligned} |r|^2 &= r_x^2 + r_y^2 + r_z^2 \\ &= l_1^2 + l_2^2 + l_3^2 + l_4^2 + 2l_1 l_2 \cos\phi_2 \\ &\quad + 2l_3 l_4 \cos\phi_4 + 2(l_1 \cos\phi_2 + l_2) \\ &\quad \cdot \{l_3 \cos\phi_3 + l_4 \cos(\phi_3 + \phi_4)\}. \end{aligned} \tag{20}$$

Suppose $\phi_3 = \phi_4$ in (20); then we obtain

$$\cos\phi_2 = \frac{|r|^2 - (l_1^2 - l_2^2 + l_3^2 + l_4^2 + 2l_3 l_4 \cos\phi_4 + 2l_2 M_2)}{2l_1 M_2}, \tag{21}$$

where

$$M_2 = l_2 - l_4 + l_3 \cos\phi_4 + 2l_4 \cos^2\phi_4.$$

Also suppose that $\phi_3 = \phi_4$ in (19) to obtain

$$\sin \phi_2 = \text{sign}\,(\psi_2) r_z / M_2. \tag{22}$$

From (21) and (22),

$$4l_1^2(M_2^2 - r_z^2) = \{|r|^2 - (l_1^2 - l_2^2 + l_3^2 + l_4^2 + 2l_3 l_4 \cos \phi_4 + 2l_2 M_2)\}^2. \tag{23}$$

Thus we find the next fourth-degree equation about $\cos \phi_4$:

$$c_1 \cos^4 \phi_4 + c_2 \cos^3 \phi_4 + (c_3 + 8|r|^2 l_2 l_4) \cos^2 \phi_4 + \{c_4 + 4|r|^2 l_3(l_2 + l_4)\} \cos \phi_4 + \{c_5 + 2|r|^2(l_1^2 + l_2^2 + l_3^2 + l_4^2 - 2l_2 l_4) - |r|^4 - 4l_1^2 r_z^2\} = 0. \tag{24}$$

In (24), $c_1, c_2, \cdots, c_4$, and c_5 are constants calculated with physical parameters l_1, l_2, l_3, and l_4 (see Appendix II). The fourth-degree equation (24) with unknown $\cos \phi_4$ is reduced to an auxiliary cubic equation by Ferrari's method [12]. This cubic equation can be solved, for example, by Cardan's formula [12]. The root which satisfies $0 \le \cos \phi_4 \le 1$ is selected as the suitable one from the four roots of the given fourth-degree equation. The suitable root is not found when the position of vector r is out of fingers' reach. If the value ϕ_4 is found, the value ϕ_2 is uniquely determined from (19). Then, the value ϕ_1 is determined from (17) or (18). Here, two values of ϕ_1 are obtained from (17) or (18). One of these is for the case where the value of ϕ_4 is positive. The other is for when the value of ϕ_4 is negative. From these results, the most suitable set of control variables is selected after testing the condition of a designated position. Then we can obtain the values of ϕ_1, ϕ_2, ϕ_3, and ϕ_4 as the finger solution.

The position of the fingertip can be precisely controlled by using the result of the finger solution. But the supporting condition between the finger and an object is not determined, because the fingertip has spherical volume in general. Therefore, if the finger is controlled only in position, the finger would be deadlocked by the friction occurring between the finger and the object. Otherwise, the object would slip and the supporting condition would be compelled to change. The slip is caused by a coercive motion of the finger, different from a slip caused by the lack of grasping power. On the other hand, a finger which has the ability to control not only its position but also its orientation can keep the object from slipping and can change the supporting condition continuously. The finger can be controlled as if it rolled its tip on the surface of the object. Therefore, if fingers of this kind are installed in the object-handling system, the system would accomplish the manual tasks stably, smoothly, and continuously. Also, in the tasks, it would become possible to maintain the relative grasping condition between the fingers and the object. For realizing this advanced control, we need to have other sets of values which keep the finger position unchanged but the orientation of the link l_4 different. These values are solved from (17), (19), and (20) by assuming that ϕ_4 takes a slightly changed value from the value of ϕ_4 which is calculated under the condition $\phi_3 = \phi_4$. Namely, the value of ϕ_4 is assumed to be the value obtained by slightly changing the initial value of ϕ_4. The value of ϕ_2 is determined by solving the quadratic equation. Then, the values of ϕ_3 and ϕ_1 are obtained. This calculating procedure ends when the orientation of link l_4 comes close to the designated orientation. The most appropriate set of values for the control variables is obtained when the value of the function T_{dev} is greater than a threshold value that is nearly equal to 1. The function T_{dev} is defined as

$$T_{\text{dev}} = \min\left\{\sum_{i=1}^{3} T_f(i,1)T'_f(i,1), \sum_{i=1}^{3} T_f(i,2)T'_f(i,2), \sum_{i=1}^{3} T_f(i,3)T'_f(i,3)\right\}. \tag{25}$$

The value of the function T_{dev} is 1 when the calculated orientation is equal to the given orientation (i.e., $T'_f = T_f$).

The rotation matrix T'_f contains three independent variables ϕ_1, ϕ_2, and $\phi_3 + \phi_4$. Therefore, the remaining degree of freedom to control the position of the fingertip is limited to only one. This is to change the value of ϕ_4 under the condition that the value of $\phi_3 + \phi_4$ is not changed. Therefore, in the case where the orientation is kept constant, the movable area of the fingertip is limited to an ellipse that is on the plane denoted by the next equation expressed in the coordinate system C_f:

$$\sin \phi_1 \sin \phi_2 X - \cos \phi_1 \sin \phi_2 Y + \text{sign}\,(\psi_2) \cos \phi_2 Z + l_1 \sin \phi_2 = 0. \tag{26}$$

The position components (X_e, Y_e, Z_e) of the center of the ellipse are written in the following form:

$$X_e = -(l_1 + l_2 \cos \phi_2) \sin \phi_1 - l_4\{\cos \phi_1 \sin(\phi_3 + \phi_4) + \sin \phi_1 \cos \phi_2 \cos(\phi_3 + \phi_4)\}; \tag{27}$$

$$Y_e = (l_1 + l_2 \cos \phi_2) \cos \phi_1 - l_4\{\sin \phi_1 \sin(\phi_3 + \phi_4) - \cos \phi_1 \cos \phi_2 \cos(\phi_3 + \phi_4)\}; \tag{28}$$

$$Z_e = \text{sign}\,(\psi_2)\{l_2 + l_4 \cos(\phi_3 + \phi_4)\} \sin \phi_2. \tag{29}$$

In the case where the constant orientation is formed when the value of ϕ_2 is 0 or $\pi/2$, the movable area of the fingertip is limited to a parabola or to a straight line on the plane denoted by (26), respectively.

When the elements of the matrix T'_f are fixed, the angle between the projected line of l_4 into the $X_f\,Y_f$-plane and the Y_f axis is written as

$$\eta = \cos^{-1} \frac{\cos \phi_1 \cdot \cos \phi_2 - \sin \phi_1 \cdot \tan(\phi_3 + \phi_4)}{\sqrt{\cos^2 \phi_2 + \tan^2(\phi_3 + \phi_4)}}. \tag{30}$$

The angle η roughly shows the orientation of the link l_4.

The control inputs for the SF and TF are determined by solving a fourth-degree equation. Those for the FF are determined by solving the quadratic equations obtained from (17), (19), and (20) on the condition $l_4 = 0$ and $\phi_4 = 0$.

The FF cannot change the orientation of the link l_3 without changing the position of the fingertip. This is caused by a deficiency in the degrees of freedom.

IV. Control Modes of the Finger Joint

There are two types of control for a joint. One uses a physical feedback loop in a servo system. According to this type, a signal is given only once to the servo system and after that the hardware is left in full charge of the processing. The other does not use the physical feedback loop: the present value of a controlled output is detected, and then a signal which makes an expected result is given to the servo system after some calculations. The former type is called a hardware servo, whereas the latter is called a software servo. In the former type, the change of the control mode requires feedback loop changing procedures. But once it is completed, there remains nothing to be done. On the contrary, the latter requires repetition of the procedure in every cycle of the servo control. In order to simplify the control, the hardware servo is adopted in our system. This control method will be explained in the following by an example of the change in mode from position to torque control.

Generally, the servo system may have some deviations, since the gain of the servo amplifier is finite. Under such a case, smooth mode changing from position to torque control without any rapid variation of the control system is impossible because an input signal is simply decided from the relation between an input and an output. In order to obtain smooth control, the deviation of the servo system must be kept to the same value before and after the change. Therefore, the initial signal $(V_i)_t$ for torque control is calculated by the following:

$$(V_i)_t = (V_i)_p - E_p + E_t, \tag{31}$$

where $(V_i)_p$ denotes the signal for the position control, E_p the feedback value in the position control, and E_t the feedback value in the torque control, just before the change from position to torque control. The error signal $(E_i = (V_i)_p - E_p)$ is temporarily held by a sample-and-hold device. In the holding duration, the feedback loop is changed from position to torque control, and the value $(V_i)_t$ calculated from (31) is added to the servo system. When all these procedures are completed, the error signal $(E_i = (V_i)_t - E_t)$ which is not changed is led into the servo system in the sample state of the sample-and-hold device. Hereafter, the signal V_i to the servo system is used not for position but for torque control. The mode change from torque to position control can be carried out in a manner similar to the one mentioned above.

In torque control based on a conventional position control circuit, the input signal to the servo system is always newly set by monitoring the controlled result of the torque. Such a software servo system requires much computation to obtain the input signal. Since the load becomes larger in a multijointed finger system, it is necessary to reduce the load as much as possible. But we have no idea of how to solve this problem. For these reasons, the hardware servo system is considered to be effective.

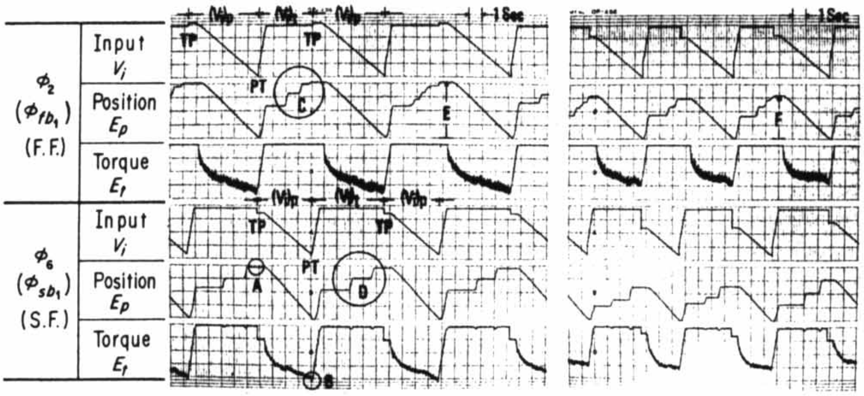
The finger motions become indeterminable if the torque control is applied to all the joints. It is necessary to apply the position control to some joints. The control modes of the joints may be determined depending on whether they move actively or passively. In the case of follow-on motion of two fingers, where one moves actively by the position control and the other moves passively by the torque control, the finger subsystem can grasp an object with the proper grasping force so as not to crush it.

In our finger subsystem, the torque of each joint, as well as its position, is easily controlled by the hardware servo. Thus the cooperative motion of the fingers can be realized by changing the control mode between the position and the torque control. The control algorithm based on cooperative motion makes it possible to keep the forces between the fingers constant, independently of their position. Thus the same kinds of tasks are accomplished with the same control algorithm, even if the dimension and the shape of the objects vary in some degree. In the experiments, the swing motion in which an object is transferred left and right is demonstrated. Each joint for degrees of freedom ϕ_2, ϕ_6, and ϕ_{10} (see Table I) is activated. But the other joints are regulated to maintain the angles corresponding to the initial grasping. The relations among the input signal V_i, the feedback value of the position E_p, and the feedback value of the motor armature current E_t which are obtained in the experiment are shown in Fig. 8. (Since degree of freedom ϕ_{10} behaves similarly to ϕ_6, the relation for ϕ_{10} is not shown.) E_p is detected by a potentiometer, and E_t is obtained from the motor armature current.

In the diagrams, (a) shows the case where a bar of 2.5-cm diameter is swung, and (b) shows where a bar of 7.5-cm diameter is swung. TP and PT represent the point where the control mode changes from torque to position control and vice versa. The time duration for the change of the control mode of the servo system is about 1.2 ms. In contrast to the period of 7.2 s for (b), that for (a) is made longer because of the wider swinging range due to the smallness of the object. As can be seen from these diagrams, the torque-controlled finger follows the position-controlled one. In this experiment, the finger subsystem has swung the bars smoothly without dropping and crushing them.

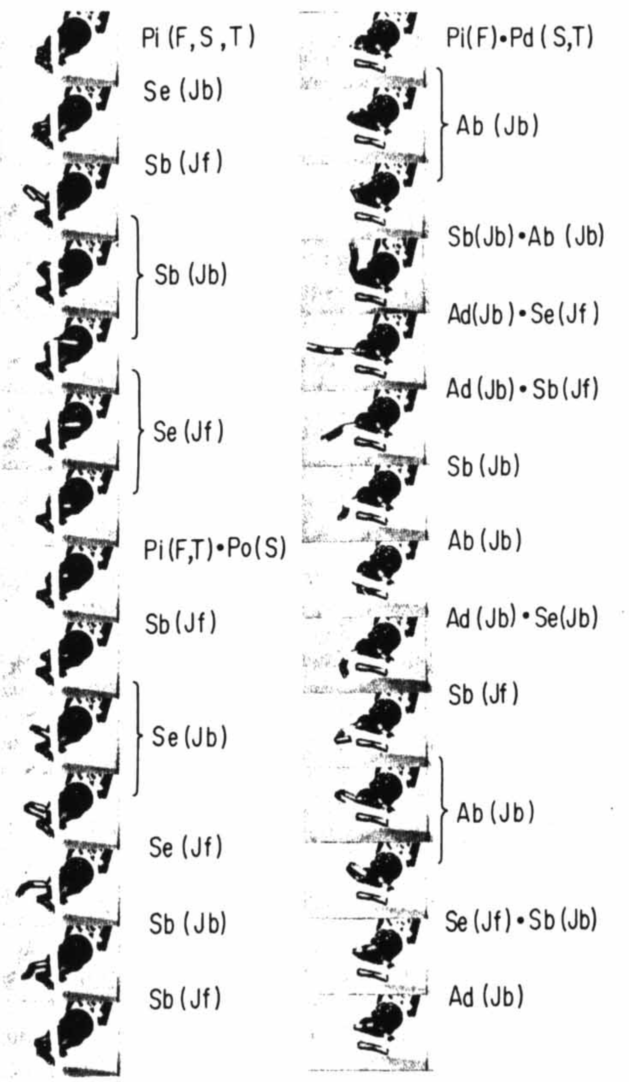
V. Experiments with Simple Manual Tasks

The finger subsystem is activated by a simple method of computer control. In this method, the computer uses a sequence of set points for each finger joint. The data of the set points have been gathered under the manual mode of position control and stored in a memory. The number of set points in the experiments is 30 (i.e., 30 patterns of finger configuration are memorized) for one manual task. Control inputs between two neighboring sets of data are generated by interpolating uniformly for all joints of the fingers. Fig. 9 shows the experimental results of the fundamental motions of the finger (SF) during bend-in (BI), bend-out (BO), the sideways motion of pressing out and extending (SOE), and the sideways motion of pressing inside with bending (SIB). On the other hand, Fig. 10 shows the results of the practical



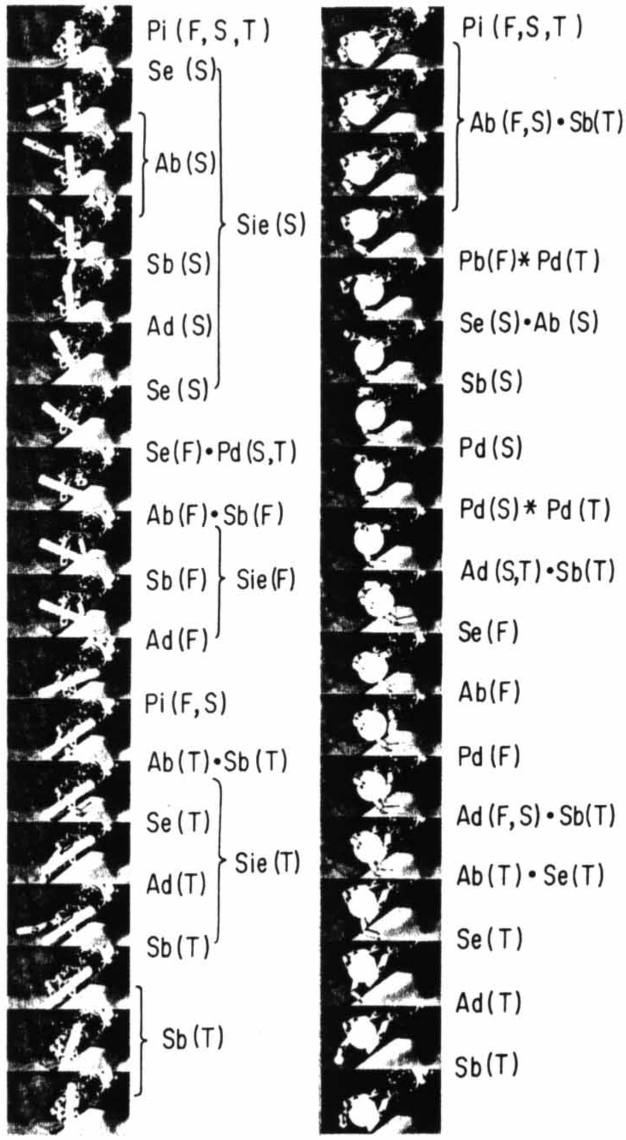
(a) (b)

Fig. 8. Experimental results of swinging an object. (a) Result when bar of 2.5-cm diameter is swung. (b) Result for bar of 7.5-cm diameter. Invariableness shown in circles *A* and *B* proves that the servo system is changed smoothly from torque to position and position to torque control, respectively. Variation shown in circles *C* and *D* proves that the finger controlled in torque follows on the finger controlled in position, maintaining the torque corresponding to the signal $(V_i)_t$. The minimum to maximum range of the signal E_p in one cycle of the swinging motion increases as the size of the object becomes large, since the swinging boundary of each finger is fixed. This property is confirmed by comparing the magnitudes of *E* and *F*.



(a) (b)

Fig. 9. Fundamental motions of finger by computer control. (a) BI and BO. (b) SOE and SIB.



(a) (b)

Fig. 10. Computer-controlled manual tasks. (a) Bar turning. (b) Sphere turning.

TABLE II
SYMBOLIC NOTATION OF FINGER MOTIONS

Symbol	Meaning
Sb	Simple bending
Se	Simple extending
Ad	Adduction
Ab	Abduction
Pi	Press inside
Po	Press outside
Pd	Press adductive side
Pb	Press abductive side
Bi	Bend in
Bo	Bend out
Sib	Sidewise motion of pressing inside with bending
Sie	Sidewise motion of pressing inside with extending
Sob	Sidewise motion of pressing outside with bending
Soe	Sidewise motion of pressing outside with extending

tasks of bar and sphere turning. These finger motions are explained with symbols (refer to Table II) which are based on the analysis [1] of human finger motions. The symbols J_f and J_b represent the joints near the tip (forward) and near the root (backward) of a finger, respectively. The form $X(Y)$ means that a joint or finger Y performs motion X. A dot $(\cdot)$ shows the simultaneous occurrence of two motions immediately before and after the dot. An asterisk $(*)$ means that two motions before and after the asterisk are performed with the prehensional form of pinching or picking. These experimental results have shown that the fingers move smoothly and perform manual tasks stably and continuously.

VI. CONCLUSIONS

An object-handling system for manual industry has been presented. The finger subsystem has a structure similar to that of a human and is characterized by the motional abilities of adduction and abduction. The kinematics of the system has been discussed, and we have found that the finger solution is given by solving a fourth-degree equation. Position and torque control have been carried out by a hardware servo system, and it has become possible to realize cooperative motions of the fingers. In the swing experiment, the force control has become useful for handling an object flexibly and for reducing the calculations for the finger solution. Several simple experimental tasks have been accomplished by computer control. It has been observed that the fingers move stably and smoothly, in spite of the fact that the fingers are driven in the position control mode only. This seems to be dependent on the human-like structure of the finger subsystem. The manual tasks could be carried out more smoothly by using a control algorithm which realizes the cooperative motion of the fingers. The driving force for the finger joint has been transmitted by cables guided in hoses. Torque control has been carried out on the assumption that the frictional force arising between the cable and the inner wall of the hose is constant, since the speed of the cable's motion is very low. It is desirable to reduce the influence of the frictional force for more flexible control.

The development of the new object-handling system is regarded as a successful step toward the automation of the manual industry. The author believes the system could be widely applied to various fields where flexible manipulation is needed. If an adequate hook is installed, like a nail in the fingertip, on which a string can be temporarily fixed, it would enable the tying of strings with a single knot. String tying is the main task in handling parts having holes, and we intend to expand our system to be able to accomplish this task. Further, as a long-range plan, several human daily tasks can be proposed, for instance, manipulating buttons and chopsticks. Research in these areas would contribute to the development of an artificial limb and an anthropomorphic manipulator.

Although the exploitation of this research may be farther off than the construction of a language for finger operation, the language is important in order to describe the manual task and to develop a programming system for computer control. Symbolic notations based on the fundamental finger motions would be useful in constructing the language. Several parameters concerning tactile sensors and force sensors may be incorporated in the description of the language.

APPENDIX I
TRANSFORMATION MATRICES

$$B_i = \begin{bmatrix} \cos\psi_i & 0 & \sin\psi_i & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\psi_i & 0 & \cos\psi_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$C_i = \begin{bmatrix} \cos\phi_i & -\sin\phi_i & 0 & -l_i\sin\phi_i \\ \sin\phi_i & \cos\phi_i & 0 & l_i\cos\phi_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

APPENDIX II
PHYSICALLY DEFINED CONSTANTS

$$c_1 = 16l_4^2(l_1^2 - l_2^2),$$
$$c_2 = 16l_3l_4(l_1^2 - l_2^2 - l_2l_4),$$
$$c_3 = 4(l_1^2 - l_2^2)(2l_2l_4 + l_3^2 - 4l_4^2) - 4l_4(4l_2l_3^2 + 2l_2l_4^2 + l_3^2l_4),$$
$$c_4 = 4l_3\{l_1^2(l_2 - 3l_4) - l_2(l_2^2 - l_2l_4 - l_4^2) - (l_2l_3^2 + l_3^2l_4 + l_4^3)\},$$
$$c_5 = -(l_1^2 - l_2^2 + l_3^2 - l_4^2)^2 + 4(l_2l_4 - l_3^2)(l_2^2 - l_2l_4 + l_4^2) - 4l_1^2l_2l_4.$$

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Pareto Optimality and Equity in Social Decision Analysis

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Abstract—**The incorporation of the preferences of different groups into a decision analysis for public-sector decisionmaking is considered. We show there is no way to do this that is guaranteed to be both efficient (in the sense that no other solution will make some group better off without making another worse off) and that considers equity (i.e., the different impacts on various groups).**

Often, decisions made by public-sector decisionmakers have differing impacts on various groups within society. Howard [5] has proposed the use of decision analysis to analyze such decision problems. Here we examine the incorporation of the preferences of different groups into a decision analysis and show there is no way to do this that is guaranteed to be both efficient (in the sense that no other solution will make some group better off without making another worse off) and that considers equity (i.e., the different impacts on various groups).

Decision analysis [4], [9] can be summarized as follows: The set of alternative decisions $\{a_1, a_2, \cdots, a_n\}$ available is described, and the uncertainty about what consequence will result from each alternative is summarized by $P_1(c), \cdots, P_j(c), \cdots, P_n(c)$, where $P_j(c)$ is the probability distribution over the uncertain consequence c, given that a_j is selected. The decisionmaker's preference for the various consequences are summarized in a von Neumann–Morgenstern utility function $U(c)$. The axioms of decision theory [8] imply the alternative a_j, which maximizes the expected utility

$$E_j[U(c)] \equiv \sum_c U(c)P_j(c) \tag{1}$$

should be selected.

In situations where a public-sector decisionmaker is faced with groups that have differing judgments about uncertainties and conflicting preferences, it is not clear what probabilities $P_j(c)$ and utilities $U(c)$ should be used in the analysis; in this correspondence, we will consider the determination of $U(c)$. It seems that in a democratic governmental system the "social" utility function that is used for the decision analysis should be a function of the utility functions of the society's citizens. That is,

$$U(c) = u[u_1(c), \cdots, u_k(c), \cdots, u_N(c)] \tag{2}$$

where u_k, $k = 1, 2, \cdots, N$, is the utility function of the kth citizen in the society and u is an unspecified function. We shall show that if U is a function of the individual preferences u_k, $k = 1, 2, \cdots, N$, as indicated in (2) then the resulting social utility function will have characteristics that may be undesirable.

Harsanyi [3], Keeney [6], and Keeney and Kirkwood [7] have also studied the aggregation of individual utility functions into a social utility function. They have investigated efficiency and equity questions similar to those considered here.

Pareto Optimality

A central concept in many studies of social decisionmaking has been Pareto optimality [2]. An alternative is Pareto optimal if there does not exist another alternative that is at least as acceptable to all society members and definitely preferred by some. The *Pareto Optimality Criterion* specifies that in any social decision problem a Pareto optimal alternative should be selected. This criterion seems reasonable since if a Pareto optimal alternative is not selected then there is another choice which will be more preferred by at least one person and not less preferred by anyone. There seems to be no reason not to select the alternative that makes someone better off. However, imposing this condition restricts the form of the social utility function as the following theorem shows.

Theorem 1: The only continuous social utility function $U(c) = u[u_1(c), \cdots, u_k(c), \cdots, u_N(c)]$, which results in decisions that obey the Pareto optimality criterion, is

$$U(c) = \sum_{k=1}^{N} \lambda_k u_k(c) \tag{3}$$

where λ_k, $k = 1, 2, \cdots, N$, are arbitrary constants greater than zero.

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